Single-Layer Channel Routing and Placement with Single-Sided Nets

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Abstract

This paper considers the optimal offset, feasible offset, and optimal placement problems for a more general form of single-layer VLSI channel routing than has usually been considered in the past. Most prior works require that every net has exactly one terminal on each side of the channel. As long as only one side of the channel contains multiple terminals of the same net, we provide linear-time solutions to all three problems. Such results are implausible if the placement of terminals is entirely unrestricted; in fact, the size of the output for the feasible offset problem may be $\Omega(n^2)$. The linear-time results also hold with a ragged boundary on the side of the channel with multiple connections to the same net.

1 Introduction

We are given two horizontal lines, whose positions may be adjusted to form the top and bottom boundaries (*sides*) of a rectilinear grid, and a set of n nets. Each net consists of *terminals* located at grid points on the two sides, and we refer to the region between (and including) the two sides as the *channel*. An acceptable routing must specify paths along grid-line segments within the channel such that terminals belonging to the same net are connected, but the wiring paths for any two different nets do not cross or have any grid-line segments in common. (See Figure 1.) We assume that there exists such a routing, a condition that can be verified in linear time [8]. The principal measure of routing quality is the number of horizontal grid lines that are used, or, equivalently, the *separation* between the sides of the channel. In seeking to minimize the separation, we allow the two rows of terminals to be shifted relative to one another by an amount referred to as the *offset* as illustrated in Figure 1. (The offset may be positive or negative.)

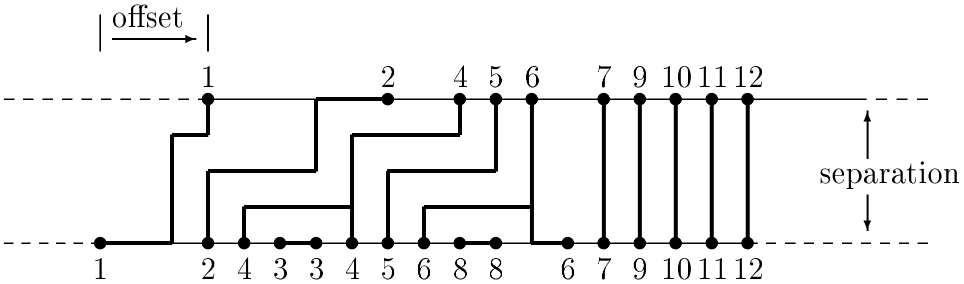

Figure 1: A single-layer channel and a routing that achieves the minimum separation. The number above or below each terminal identifies the net to which it belongs.

This situation models connection of VLSI modules having terminals on their boundaries. Though VLSI chips generally use more than one interconnection layer, single-layer routing actually becomes more relevant

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as technological advances increase the number of layers on a chip. The heuristic multilayer channel router MulCh [1] obtains good results by dividing general problems into subproblems with one, two, or three layers.

In this paper, we consider three specific problems relating to single-layer channel routing. The *optimal offset problem* involves finding the offset that minimizes the amount of separation necessary to route the channel. The *feasible offset problem* involves finding all offsets that give enough room to route at a given separation. Finally, the *optimal placement problem* considers a scenario in which the terminals on each of the two sides are grouped into several chunks. Within each chunk, the positions of the terminals are fixed, but, on each side of the channel, the chunks can slide back and forth as long as their order does not change. The goal in this problem is to minimize the channel length given a channel width.

The *river routing* scenario, in which each net has exactly one terminal on each side of the channel has been well analyzed [4, 6], but real channels may include nets with many terminals (e.g. some of the examples in [1]). (Such multiple connections are even more likely in the optimal placement problem with multiple modules.) We show that if only one side of the channel contains multiple connections to the same net, then feasible offset, optimal offset, and optimal placement can still be solved in $O(n)$ time for problems with n nets. (The necessary premise is relatively likely to be satisfied by some one-layer subproblems of a multilayer problem if not by the full set of nets.) On the other hand, if the terminal positions are entirely unrestricted (except for the planarity requirement), linear-time solutions are implausible. In particular, there may be $\Omega(n^2)$ disjoint intervals of offsets that are feasible for a given separation. Thus, the feasible offset problem cannot be solved in better than $\Omega(n^2)$ time, except perhaps by using some unusual output representation; furthermore, the optimal offset and optimal placement problems do not appear to be easier even though they have a smaller output size. (For further analysis of this unrestricted version of the problem, see [3].)

There is actually no loss of generality in restricting attention to nets that have just two terminals (by a reduction described in [2] that derives from "folklore"). Thus, river routing is overly restrictive only in that it requires that the two terminals must be on opposite sides of the channel. We refer to the type of net allowed in river routing as a *two-sided net*, whereas a net with its two terminals on the same side of the channel is a *single-sided net*.

We show in Sections 3, 4, and 5 of this paper that the feasible offset, optimal offset, and optimal placement problems can all be solved in $O(n)$ time as long as all single-sided nets are on one side of the channel. The results also apply when the channel boundary containing single-sided nets is ragged. These results depend upon the convenient expression in Section 2 of the routability conditions for such a channel.

2 Cut Conditions

In this section we use the theory of single-layer routing developed by Maley [5] to derive a routability test for channels with single-sided nets on one side. We justify this routability test carefully, since the literature contains *erroneous* proposals of a similar test for general channels (as explained in [2]).

Without loss of generality, we assume all single-sided nets are on the bottom side. The *contour* of these single-sided nets is the routing boundary that the two-sided nets must stay unit distance away from when the single-sided nets are routed as tightly as possible against the bottom of the channel. (See Figure 2.)

Our first step for all the problems treated in this paper is to determine the contour of the single-sided nets, which Pinter [7] shows can be done in linear-time:

Lemma 1 (Pinter) *The bendpoints in the contour of a set of n single-sided nets can be found in $O(n)$ time.* ■

We need a few more definitions to invoke Maley's theory. A *cut* χ is a line segment connecting a top and bottom terminal or traveling at 45° from a terminal to the opposite side of the channel; these correspond to the "pivotal cuts" in [5]. The *flow* across χ is the number of nets that must cross χ, i.e., those nets having terminals on both sides of χ or having an endpoint of χ as a terminal. The *capacity* of χ is one greater than the maximum of the horizontal and vertical separations of its endpoints; if χ runs from (x_1, y_1) to (x_2, y_2),

$$capacity(\chi) = max\{|x_1 - x_2|, |y_1 - y_2|\} + 1 \ .$$

The cut χ is *safe* if $flow(\chi) \leq capacity(\chi)$, which means that there is enough space along χ for the wires to get through.

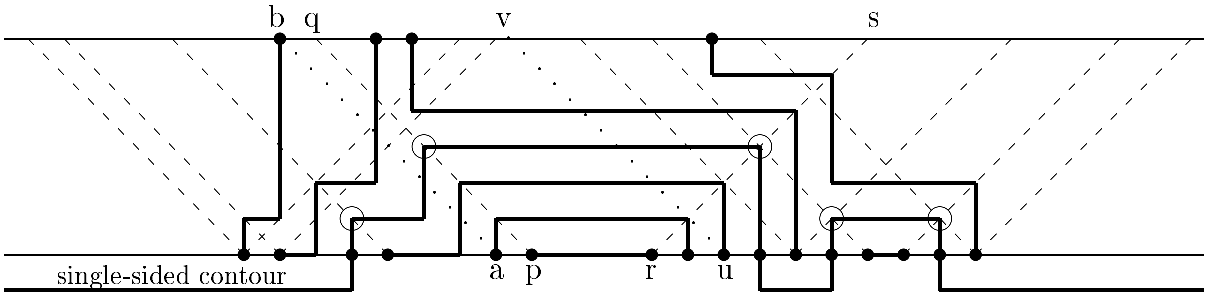

Figure 2: Only those 45° cuts from bottom terminals shown with dashed lines need be checked. The hollow circles mark the convex corners of the contour of single-sided nets.

Lemma 2 *A channel is routable if and only if every cut is safe.* ■

This lemma follows from the corresponding result in [5, §2.1]. (Our slightly different definitions of flow and capacity and routing on channel boundaries allow Maley's formulation in terms of cuts emanating from "feature" endpoints to correspond to cuts emanating from terminals.)

We now show that many cuts can be removed from consideration. First, we need not check cuts emanating from a terminal a that are outside the "cone" formed by the two 45° cuts from a; pivoting a 45° cut around a so that the other endpoint moves further away increases capacity at least as much as flow, so cuts outside the cone are safe if the 45° cuts are. Now we say a cut χ is *dominated* by cuts α and β if all these cuts have the same capacity, and the nets that must cross χ must all cross α or all cross β. In this case, χ need not be checked because it is unsafe only if α or β is unsafe. Since all the single-sided nets are on the bottom, many cuts are dominated by others. In particular, any cut χ connecting to a terminal only at the top of the channel is dominated by the two parallel cuts emanating from bottom terminals, one to the left and one to the right, that are nearest to χ. Additionally, any remaining cut that is not a 45° cut is dominated by the two 45° cuts emanating from its bottom endpoint. Finally, the cuts from terminals of single-sided nets are unnecessary unless they cross the contour of single-sided nets at a convex corner. For example, in Figure 2, cut $\overline{ab}$ has no greater flow than $\overline{pq}$, and $\overline{uv}$ is dominated by $\overline{pq}$ and $\overline{rs}$. From the above reasoning, we have:

Theorem 3 *A channel with all single-sided nets on the bottom is routable if and only if all* 45° *cuts from bottom terminals of two-sided nets and all* 45° *cuts crossing the contour of single-sided nets at a convex corner are safe.* ■

We need a few more definitions to express the safety conditions algebraically. Let $b_1, b_2, \ldots, b_t$ denote the x-coordinates of the upper terminals of two-sided nets (in sorted order) and $a_1, a_2, \ldots, a_t$ denote the x-coordinates of the bottom terminals of two-sided nets; a_i is to be connected to b_i, for $1 \leq i \leq t$. Also let $c_1, c_2, \ldots, c_k$ denote the x-coordinates where the contour of single-sided nets bends. Then let t_i be the number of two-sided nets whose bottom terminals are to the left of c_i, and let e_i be the extension of the single-sided contour at c_i, the nonnegative distance that the contour rises above its baseline at that column.

Theorem 3, leads directly to a set of $2(t+k)$ conditions for feasibility of separation s and offset d. But it is helpful for Section 4 to perform a further analysis to bring the number of conditions to at most $4t$. (To simplify our presentation, we assume through the rest of this paper that a separate check is performed to ensure that $e_i \leq s+1$ for all i.) We do this by deriving the tightest constraint on each b_i, considering separately four classes of cuts based on whether the cut is from a terminal of a two-sided net or runs through a convex contour corner, and whether the cut is angled to the left or the right. By defining c_{r_i} to be the nearest bendpoint to the right of a_i such that $t_{r_i} - i + e_{r_i} > s$ and c_{l_i} to be the nearest bendpoint to the left of a_i such that $i - t_{l_i} + e_{l_i} > s+1$, the complete set of constraints is

$$max\{a_{i-s-1}, c_{l_i} - e_{l_i}\} + s < b_i + d < min\{a_{i+s+1}, c_{r_i} + e_{r_i}\} - s \qquad 1 \leq i \leq t\ . \tag{1}$$

Here we define $a_j = -\infty$ if $j \leq 0$ and $a_j = \infty$ if $j > t$; also c_{l_i} (c_{r_i}) is defined to be $-\infty$ (∞) if there is no bendpoint satisfying the necessary conditions. The analysis can be extended to the case in which the bottom boundary of the channel is ragged, i.e., bottom terminals of two-sided nets may also have extensions, but we omit that case for simplicity.

3 The Feasible Offset Problem

If we let

$$l(s) = \max_{1\le i\le t}\{a_{i-s-1}, c_{l_i} - e_{l_i}\} + s - b_i \quad \text{and} \quad u(s) = \min_{1\le i\le t}\{a_{i+s+1}, c_{r_i} + e_{r_i}\} - s - b_i \ ,$$

then we know from Condition (1) that a pair (s, d) is feasible if and only if $l(s) < d < u(s)$.

Theorem 4 *The feasible offset problem can be solved in $O(n)$ time.*

Proof. All we need to do is to compute $l(s)$ and $u(s)$. We can find all t_i's, for $0 \le i < k$, by a linear scan. Furthermore, since l_i is nondecreasing as i increases, we can find all the l_i's in $O(n)$ time. Thus, we can compute $l(s)$ in $O(n)$ time. Similarly, $u(s)$ is computable in $O(n)$ time. (Note that once the l_i and r_i values are known, only $O(t)$ time is required.) ■

4 The Optimal Offset Problem

In this section, we use a halving technique as in [6] to solve the optimal offset problem in $O(n)$ time. We actually focus here on finding optsep(P), the minimum separation attainable with an optimal offset for the routing problem P; once optsep(P) is determined, the solution of the feasible offset problem can be used to determine the optimal offsets. From the original problem P, we create a simpler problem P^e that has about half the separation of P. The basic idea is to halve the extensions of the contour of single-sided nets, remove every other two sided net, and compact the channel horizontally to eliminate the freed space. More precisely, we perform the transformation specified as follows:

$$b_i^e = b_{2i} - i \ , \quad a_i^e = a_{2i} - i \ , \quad r_i^e = r_{2i} \ , \quad \text{and} \quad l_i^e = l_{2i} \qquad 1 \le i \le \lfloor t/2 \rfloor$$

and

$$c_j^e = c_j - \lceil t_j/2 \rceil \ , \quad \text{and} \quad e_j^e = \lfloor e_j/2 \rfloor \qquad j \in \{\, r_i^e, l_i^e \mid 1 \le i \le \lfloor t/2 \rfloor \,\} \ .$$

The following lemma states the relationship between optsep(P) and optsep(P^e):

Lemma 5 *Let* $s = \mathrm{optsep}(P)$ *and* $s^e = \mathrm{optsep}(P^e)$*. Then* $2s^e \le s \le 2s^e + 3$.

Sketch of proof. A cut χ in P that crosses f nets, p of which are single-sided nets and q of which are two-sided nets can be seen to correspond to a cut χ^e with the following properties: (1) The flow of χ^e is in the range $[\frac{p-1}{2} + \frac{q-1}{2}, \frac{p}{2} + \frac{q+1}{2}] = [\frac{f}{2} - 1, \frac{f}{2} + \frac{1}{2}]$, and (2) the horizontal extent of χ^e is diminished relative to χ to the same extent as the flow. Thus $s^e + 1 \in [\frac{s+1}{2} - 1, \frac{s+1}{2} + \frac{1}{2}]$, i.e., $2s^e \in [s-3, s]$. ■

Theorem 6 *The optimal offset problem can be solved in $O(n)$ time.*

Proof. We compute the contour of single-sided nets and the l_i and r_i values once up front in $O(n)$ time, and let $T(t)$ be the remaining time to find optsep(P), where P has t two-sided nets. In $O(t)$ time, we can transform P to P^e, which we solve recursively. From Lemma 5, once we know optsep(P^e), we only need to check 4 possible separations to achieve the optimal offset. Each separation can be checked in $O(t)$ time according to the proof of Theorem 4. Thus, we have $T(t) \le T(\lfloor t/2 \rfloor) + O(t)$, which yields $T(t) = O(t)$. ■

5 The Optimal Placement Problem

The optimal placement problem is defined in [4] as follows. The terminals on each side of the channel are grouped into chunks which must be placed as a unit. On each side of the channel, the order of the chunks is fixed, but their positions are not. As shown in Figure 3, the separation is the vertical distance between the two lines of terminals, and the spread is the horizontal dimension of the channel. Given a separation s, we

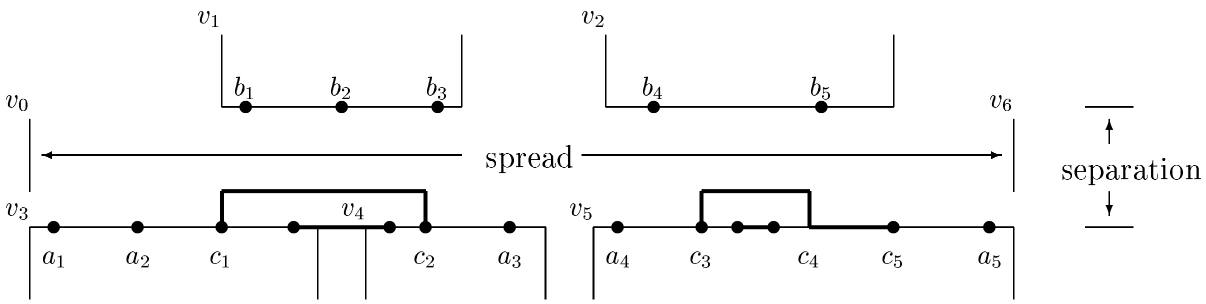

Figure 3: Two sets of chunks on either side of a channel. Variables are assigned to the horizontal position of each chunk's left edge.

seek a placement which achieves the minimum spread. For simplicity, we assume that terminals can not sit on the corners of the chunks; removing this restriction forces only slight modification to the algorithm.

From Section 2, the condition for the channel to be routable is

$$\alpha_i + s + 1 \leq b_i \leq \beta_i - s - 1, \qquad 1 \leq i \leq t\ , \tag{2}$$

where $\alpha_i = max\{a_{i-s-1}, c_{l_i} - e_{l_i}\}$, and $\beta_i = min\{a_{i+s+1}, c_{r_i} + e_{r_i}\}$. The key observation here, is that every α_i, β_i, and b_i value corresponds to a fixed position on some module, independent of the module placement. Thus, we can translate our cut conditions (2) into constraints on the module placement in the same fashion as in [4]. Let us number the chunks from 1 to k on the top and $k+1$ to m on the bottom. For each chunk i, let the variable v_i represent the horizontal position of its left edge. Any placement can therefore be specified by an assignment of values of these variables. Also add two variables, v_0 and v_{m+1}, to the set of variables to represent the left and right boundaries of the channel. The spread is thus $v_{m+1} - v_0$.

Now, the constraint $\alpha_i + s + 1 \leq b_i \leq \beta_i - s - 1$ can be rewritten in the form $v_g - v_f \geq r_{gf}$ and $v_h - v_g \geq r_{hg}$. For each pair of adjacent chunks i and $i+1$ on the same side, there is an additional constraint $v_{i+1} - v_i \geq w_i$, where w_i is the width of chunk i. We also have constraints $v_1 - v_0 \geq 0, v_{k+1} - v_0 \geq 0, v_{m+1} - v_k \geq w_k$, and $v_{m+1} - v_m \geq w_m$ for boundary conditions.

Now define a *placement graph* $G(V, E)$ to be a directed graph such that each vertex represents a variable v_i, and a directed edge with weight λ_{fg} goes from v_f to v_g if there is a constraint of the form $v_g - v_f \geq \lambda_{fg}$. Minimizing $v_{m+1} - v_0$ subject to the constraints can be achieved by solving a single-source-longest-paths problem in the placement graph. Furthermore, It is shown in [4] that if a placement graph satisfies Lemma 7 below, linear time suffices to solve the necessary longest-paths problem. Thus, we need only show Lemma 7 holds despite having generated our constraints from a more general arrangement of nets than in [4].

To state Lemma 7, we define a partial order $\prec$ on the vertices so that $u \prec v$ when the chunks corresponding to u and v lie on the same side of the channel and u's is to the left of v's. The boundary vertices v_0 and v_{m+1} satisfy $v_0 \prec x \prec v_{m+1}$ for all other vertices x. The partial order $\preceq$ is the natural extension to $\prec$ that includes equality. Also, a *cross* edge is an edge corresponding to chunks on opposite sides of the channel.

Lemma 7 *Any placement graph G=(V,E) has the following properties:*
(1) There do not exist cross edges (u,v) and (x,y) such that $u \prec x$ *and* $y \prec v$.
(2) There do not exist cross edges (u,v) and (x,y) such that $v \prec x$ *and* $y \prec u$.

Proof. Note first that α_i and β_i are nondecreasing as i increases. (To see this for α_i, just note that a_{i-s-1} and $c_{l_i} - e_{l_i}$ are nondecreasing.) Now we prove properties (1) and (2) by contradiction.
(1) There are two cases for a pair of edges violating property (1), each of which yields a contradiction:

Case I: The edge (u, v) is caused by b_i in v and α_i in u, and the edge (x, y) is caused by b_j in y and α_j in x. Since $y \prec v$, b_j is to the to the left of b_i, i.e., $j < i$. Then, since α_i is nondecreasing as i increases, we have $\alpha_j \leq \alpha_i$, which implies $x \preceq u$.

Case II: The edge (u, v) is caused by b_j in u and β_j in v, and the edge (x, y) is caused by b_i in x and β_i in y. By a similar argument to Case I, we can get a contradiction.

(2) There are also two cases for a pair of edges violating property(2); again each case yields a contradiction.

Case I: The edge (u, v) is caused by b_i in v and α_i in u, and the edge (x, y) is caused by b_j in x and β_j in y. Since $v \prec x$, b_i is to the left of b_j, i.e., $i < j$. Then, since β_i is nondecreasing as i increases, we have $\beta_i \leq \beta_j$. Also $\alpha_i \leq \beta_i$ by definition; therefore, $\alpha_i \leq \beta_i \leq \beta_j$, which implies $u \preceq y$.

Case II: The edge (u, v) is caused by b_j in u and β_j in v, and the edge (x, y) is caused by b_i in y and α_i in x. Using a similar argument as in Case I yields a contradiction. ■

As indicated above, the proof of Lemma 7 immediately yields the main result of this section:

Theorem 8 *The optimal placement problem can be solved in $O(n)$ time.* ■

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